

# A fast hybrid approach to model roughness effects for the 2D tropospheric maritime long-range propagation

Thomas Bonnafont<sup>1\*</sup>, Othmane Benhammouch<sup>2</sup>, Ali Khenchaf<sup>1</sup>

<sup>1</sup>Lab-STICC UMR CNRS 6285, ENSTA Bretagne, 29806 Brest, France

<sup>2</sup>International University of Casablanca, Bouskoura 50169, Morocco

\*corresponding author, E-mail: thomas.bonnafont@ensta-bretagne.fr

## Abstract

This paper studies the long-range tropospheric electromagnetic wave propagation over the sea. An asymptotic model based on the parabolic wave equation is considered. This latter is solved using a fast and memory-efficient wavelet-based method. The roughness effects of the sea are introduced through a hybrid approach. Numerical experiments (in X-band) are provided to highlight the advantages of the wavelet-based method in the context of maritime propagation.

## 1. Introduction

Modeling the propagation of electromagnetic waves in a maritime environment is a problem of major interest for many applications, such as the optimization of coastal radar or the direct model of refractivity from clutter [1]. In this context, many effects can influence the propagation such as the refraction in the troposphere, or the sea state. Computational efficient methods are therefore needed to obtain the propagation over large distances.

The Parabolic Wave Equation (PWE) [2] is commonly used for this purpose. Indeed, this asymptotic method models the forward propagation allowing wide steps in the propagation direction. A convenient way to solve the latter is the Split-Step Fourier (SSF) [2] method that computes the propagation in two steps. First, the field is propagated in free-space in the spectral domain. Second, characteristics of the environment, such as refraction or relief, are considered in the space domain [2]. Ground composition is also incorporated through different methods [2]. In order to tackle the problem of the sea surface, a hybrid approach has been proposed for SSF [3].

Recently, a wavelet-based method, Split-Step Wavelet (SSW) [4, 5], has been proposed to solve the PWE. The method follows the same steps as SSF, but the free-space propagation is performed in the wavelet domain instead of the Fourier domain. Indeed, it has been shown that this method performs better in terms of memory and computation time efficiency [5].

The purpose of this paper is to introduce the sea's roughness effect in 2D SSW through the hybrid approach of [3], aiming at a complete deterministic and efficient method for the tropospheric long-range.

The remaining of this summary is organized as follows. Section 2 introduces the model and the discretization. Section 3 is devoted to a brief description of the SSW method and the introduction of the hybrid approach to consider the sea surface. Section 4 shows numerical experiments in the X-band. Section 5 concludes the paper.

## 2. Parabolic wave equation model and discretization

### 2.1. The parabolic wave equation

Throughout the paper, an  $\exp(j\omega t)$  time dependence, with the angular frequency  $\omega$ , and a slowly varying refractive index  $n$  are assumed.

Since it can consider the effects of the refraction, the relief, and the ground composition, the PWE model is adapted to our context. Besides, it is less burdensome in terms of boundary and mesh conditions, allowing wider steps in the propagation direction; thus, efficient schemes in terms of computation time. Taking into consideration the forward propagation, the Helmholtz equation is reduced to the wide-angle PWE [2] as follows

$$\frac{\partial u}{\partial x} = -j \left( \sqrt{k_0 + \frac{\partial^2}{\partial z^2}} - k_0 \right) - jk_0 (n^2 - 1) u, \quad (1)$$

with  $x$  the propagation direction,  $k_0$  the free-space wavenumber and  $u$  the reduced field. The latter is only true in a paraxial cone of about  $30^\circ$ .

In this article, we aim at solving equation (1) in an efficient way while accounting for the effects of the sea surface.

### 2.2. Discretization

The considered domain is of size  $[0, x_{\max}]$  and  $[0, z_{\max}]$ . For obvious numerical reasons, this latter is discretized as follows. First, a sampling of  $N_z$  points along the  $z$ -direction is performed with  $\Delta z = z_{\max}/N_z$  the step size. At a position  $x$  the field is, thus, denoted by  $u_x[p_z]$  with  $p_z \in \{0, \dots, N_z\}$ . Second, a discretization along  $x$  is applied with  $N_x$  points. The step is denoted by  $\Delta x$ .

### 3. Split-step wavelet above a rough sea surface

#### 3.1. A brief overview of split-step wavelet

Split-step wavelet is an iterative computational scheme to compute the reduced field marching in on distances. This latter follows the same steps as SSF, but the free-space propagation is performed in the wavelet domain instead of the spectral domain. It can be summarized with the following equation

$$u_{x+\Delta x} = LRW^{-1}PC_{V_s}Wu_x, \quad (2)$$

where  $W$  and  $W^{-1}$  denote respectively the fast wavelet transform and its inverse,  $C_{V_s}$  a compression operator with hard threshold  $V_s$ ,  $P$  the compressed wavelet-to-wavelet propagator described in [5],  $R$  the phase screen operator that accounts for the refraction effects, and  $L$  the operator that accounts for the relief. The relief is considered through a staircase model [2]. The ground composition is accounted for using the local image method [4], which allows modeling the ground effects with a reduced number of points.

The overall complexity of the method has been shown to be lower than the complexity of SSF, if a good compression is performed. Besides the compression error is managed through a theoretical formula [6].

#### 3.2. Propagation above the sea

To model the propagation above a rough sea surface, the hybrid approach proposed in [3] is introduced in SSW.

The idea of the method is to take both into account the effects of the roughness and of the geometry of the surface. To do so, the Elfouhaily sea spectrum [7] is cut into two. The lowest part of the spectrum is used to compute the surface, while the highest part allows to compute a new roughness parameter, which is introduced in the Fresnel coefficient [3]. The cutoff parameter of the spectrum is defined from the mesh size in the propagation direction as

$$k_{\max} = N_x \frac{2\pi}{x_{\max}}. \quad (3)$$

This technique allows to model more precisely the effects of the sea. Also, Monte Carlo simulations are performed since the sea surface geometries are randomly generated. Therefore, incorporating this method in a fast propagation scheme, such as SSW, allows for more simulations in a fewer time.

### 4. Numerical experiments

In this section, numerical experiments are presented to show that the method works well and highlight its advantages. The following results are computed in X-band (9 GHz). The source is a complex source point (CSP) of width  $W_0 = 5$  m placed at 10 m above the ground and  $z_s = -50$  m. The propagation is computed in a domain of size  $[0, 60]$  km along  $x$  and  $[0, 128]$  m along  $z$  with a mesh

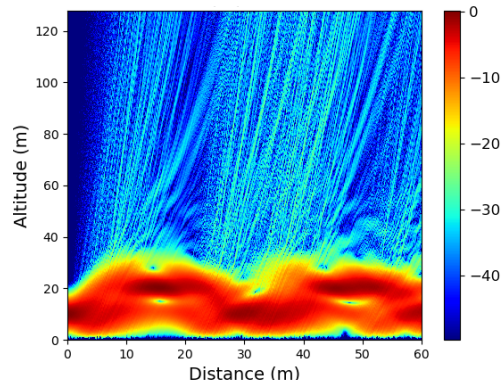


Figure 1: Normalized field  $u$  (dB) obtained with SSW

size of  $\Delta x = 40$  m and  $\Delta z = \lambda = 0.03$  m. An atmospheric duct of 40 m, modeled by a tri-linear profile of atmosphere, and a wind speed of 10 m/s are also considered. Furthermore, the sea dielectric parameters are as follows:  $\epsilon_r = 80$  and  $\sigma_r = 5$  S/m.

For the wavelet parameters, we use the symlet family with 6 vanishing moments and a maximum level of decomposition of  $L = 3$ . Also the thresholds for SSW are computed using the theoretical formula of [6] so as to obtain a maximum error of  $-20$  dB.

The normalized field obtained with SSW is plotted in Figure 1.

Figure 1 shows both the effects of the rough sea, since the geometry induces interferences, and of the tropospheric duct. The error with SSF is of order  $-30$  dB and in line with [3]. One can also note the shadow areas on the sea. Simulations on the antenna altitude could be performed to optimize the antenna position so as to have a better radiation patten to detect target on the sea.

### 5. Conclusions and perspectives

In this summary, the hybrid approach of [3] has been introduced in SSW, allowing to obtain a fast and memory-efficient method for long-range propagation in 2D.

Future works include a two-way version of the method for the propagation over different terrains, such as an island or with a ship. Besides Monte-Carlo simulations are needed in order to characterize the effects of the sea on the propagation medium. Furthermore, numerical experiments on the antenna position should be performed to optimize its altitude. Also, with the generalization of SSW in 3D [8, 9], including this hybrid approach would be an evolution of this work.

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